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PRIMARY TEACHER TRAINEES' SELF ASSESSMENT OF THEIR MATHEMATICAL SUBJECT KNOWLEDGE

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Near the beginning of their course, primary trainees completed a subject knowledge self-audit and consulted specimen answers with supporting commentary. This enabled them to assess their own responses, clarify some areas and identify areas for future study. They then completed a self-report in which they rated their responses, and added general comments about their subject knowledge. This paper presents an analysis of responses from the self report which identifies items on which students generally felt secure or insecure, reveals levels of confidence and attitudes towards mathematics and discusses the responses of those students who expressed concern about their subject knowledge.

INTRODUCTION

Prospective primary school teachers in England and Wales need to have certain minimum qualifications in English and Mathematics i.e. grade C in the General Certificate of Secondary Education or its equivalent, before entry to a course of teacher training. Concern with their subject knowledge could therefore be seen as a lack of confidence in these national qualifications or a deficit view of the students who apply for teacher training. However, students with this minimum qualification may have attained it several years ago when syllabuses were different, or may have been entered for tiers with less curriculum content, or managed to attain the grade without consistency across all aspects of the curriculum. So there may well be gaps in their knowledge, which could be of significance to the primary curriculum. Additionally, the knowledge required to meet the public qualification standard, even if it is judged to be very good by that standard, may need to be transformed and enriched in order to support the act of teaching.

The SKIMA (Subject Knowledge in Mathematics) group is a collaboration between researchers in the Universities of Cambridge, Durham, York and the Institute of Education at the University of London. It grew out of a common interest in primary teacher trainees' subject knowledge in mathematics predating the introduction of the government's (DfEE, 1998) National Curriculum for Initial Teacher Training. This required teacher training institutions in England and Wales to audit subject knowledge and where ‘gaps’ where found to make sure these were ‘filled’ by the end of the training course.

Building upon previous work (Rowland, Martin, Barber and Heal, 2000; Goulding and Suggate, 2001) the researchers devised a common procedure for use with over 400 primary trainees at Cambridge, Durham and London. It involved an early self-audit, a period when specific teaching was given and/or students could follow up areas of weakness, an audit taken in formal conditions and a follow up period when peer teaching was used. The process was designed to yield research data, which would give further insights into students’ strengths and weaknesses and their feelings about the process. This paper concentrates on the early part of this process in which students made self assessments of their mathematical knowledge. The analysis reveals aspects of mathematics on which the students generally felt secure or insecure together with insights into their confidence and attitudes towards mathematics. It also makes some comparisons between this self assessment and the performance on the formal audit.

Subject Knowledge

The conceptualisation of subject knowledge and its relation to teaching which informed the project has been detailed extensively elsewhere (Goulding, Rowland and Barber, 2002). For the purposes of the self-audit and the audit, Shulman’s construct of *subject matter knowledge* ‘the amount and organisation of the knowledge per se in the mind of the teacher’ (Shulman, 1986, p9) later analysed further (Shulman and Grossman, 1988) into *substantive knowledge* (the key facts, concepts, principles and explanatory frameworks in a discipline) and *syntactic knowledge* (the rules of evidence and proof within a discipline) were influential. Clearly we were constrained into using the list of content to be audited as set out by the government (DfEE, 1998) but we were able to devise our own questions for both the self-audit and the audit. The DfEE content included concepts not in the primary curriculum but claimed to be relevant to it. For example, although primary children would not be expected to cover the equation of the straight line, probing students’ understanding of this equation could reveal their appreciation of relationships between the symbols and the graph. It could be argued that being able to do this would help when students were using different representations and the connections between them with primary children. Similarly, the seeds of proof and algebra are now required to be sown in the primary curriculum. Students, therefore, need to be able to demonstrate the ability to reason rigorously and to see the importance of this, and to be able to express generality which is fundamental to algebra. Elsewhere (Goulding et al 2002) we have highlighted weaknesses identified in substantive knowledge but also the particular difficulties which students in previous cohorts had with generalisation and proof. We interpreted these difficulties as a weakness in syntactic knowledge, an inability or unwillingness to make and test conjectures by personal investigation.

Carol Aubrey (1997) argues for 'the central importance of disciplinary knowledge to good elementary (primary) teaching' (p.33) based on an extensive review of the literature and a close-grained observational study of four primary teachers. The Effective Teachers of Numeracy Project at Kings' College (Askew et al., 1997), however, used a much larger sample and also measured pupil progress by test score gains. In this study, the most effective teachers were those who believed in the potential of all pupils to become numerate and also had 'knowledge and awareness of conceptual connections between the areas which they taught' (p.3). These 'connectionist' teachers did not necessarily hold more advanced qualifications in mathematics e.g. 'A' level and beyond, but they were more likely to have gone on extended continuing professional development courses. With previous cohorts, members of the SKIMA group, Rowland et al. (2000, 2001) had found an association between the subject knowledge audit score and teaching performance. In particular, students with low audit scores were more likely than other students to be assessed as weak numeracy teachers. This does not necessarily contradict the Kings' study, since the audit may have been assessing students' current knowledge of mathematics more directly related to primary school mathematics. 'Connectedness' may have been elicited better by the audit than the school examinations taken some years earlier.

In our previous research we had not investigated the students' affective response to the auditing processes systematically. Field notes had revealed some anxieties in the taught sessions, and we were concerned that having a formal audit might encourage students and tutors to resort to instrumental approaches in order to get over this particular hurdle in the training year. Nonetheless, we had found the audit a useful device to 'surface and challenge' (Ball, 1990) students' assumptions about mathematics, although we were aware that there was limited time to do this effectively and that some students would feel too threatened to engage fully. Nonetheless, our previous practice which encouraged peer teaching and interviews with tutors where students talked through errors and misconceptions obliged students to think more explicitly about their mathematics. We were encouraged to strengthen this aspect of the process and subject it to more scrutiny. To access the students' affective response to the auditing process and to see how they would assess their own strengths, weaknesses and strategies for improvement self assessment was incorporated into the common process for auditing the subject knowledge of the primary PGCE students at Cambridge, Durham and London.

METHOD

Each institution included in their course documentation

1. Information and guidance on preparation for the audit, including reference to published materials.
2. A 'syllabus' listing the kinds of knowledge, skill and understanding being assessed.
3. Self-audit items (21 items) plus specimen answers and commentary.
4. A self reporting form.

Early in the course, all trainees undertook the self-audit in their own time. They were asked to complete it first, then turn to the specimen answers and accompanying commentary. This gave explanations and clarifications, and enabled the students to assess their responses. They then completed a self report form summarising their judgement of their responses to each item using a five point code:

0 I couldn't begin this question without help
1 I attempted this item, but didn't make much progress
2 I made some progress but with significant errors and omissions
3 My response was basically secure, with only minor errors and omissions
4 My response was basically secure

At the end of the form they were asked to 'add any general comments about your mathematical subject knowledge that may be of help to your tutor'. Not all students added comments. The table below gives details of the self report forms from each institution.

Institution	No of Self-audit Reports	No of Reports With comments
Cambridge	141	77 (55%)
Durham	89	41 (46%)
London	202	156 (77%)
All	432	274 (64%)

This form was designed to give quantitative data, focused very specifically upon the items of the self-audit, but also giving qualitative data from students' free response to the last request. In this way, we intended to address the two aims of:

1. Helping students and tutors build upon the self-audit in preparation for the later audit.
2. Collecting complementary kinds of research data.

The quantitative data was analysed statistically. The analysis of the students' comments was done by reading the comments, drawing up categories and coding responses according to the categories. This was done by a research assistant for the Durham cohort first and the categories and coding were then modified after discussions between the researcher and the assistant. The refined categories were then used to code the Institute and Cambridge data, and slight modifications were again made after discussions at the end of this process.

ANALYSIS

Overall self-assessment ratings

Asking students to rate their responses for each question gave data about their rating on the whole self-audit and about individual questions. For instance, the distribution for each institution was different, with Cambridge relatively symmetrical, and London and Durham skewed more towards higher self ratings. The distribution as a whole is skewed more towards the higher ratings.

Self Rating Score	London	Cambridge	Durham	Total
0-30	1 (0.5%)	3 (2.1%)	3 (3.4%)	7 (1.6%)
31-40	7 (3.5%)	17 (12.1%)	2 (2.2%)	26 (6.0%)
41-50	23 (11.4%)	32 (23.7%)	12 (13.5%)	67 (15.7%)
51-60	55 (27.2%)	38 (27.7%)	27 (30.3%)	120 (28.0%)
61-70	73 (36.1%)	29 (20.6%)	29 (32.6%)	131 (30.3%)
71-80	39 (19.3%)	20 (14.9%)	16 (18.0%)	75 (17.6%)
81-84*	4 (2.0%)	2 (0.7%)	0	6 (1.2%)
	202	141	89	432

*84 was the maximum self rating since there were 21 items each with a maximum rating of 4.

Self assessment of individual items

(a) Items with low self ratings

Inspection of individual items revealed only slight differences between the institutions. The four questions with markedly lower ratings were

- A number operations question involving the terms commutative and associative
- A graphical question involving the gradient of a straight line
- A shape and space question involving transformations, congruence and similarity
- A shape and space question involving constructions, Pythagoras and ratio

It seems very likely that the terminology in the first three questions was a source of difficulty.

(b) Items with high self ratings

The four questions with markedly higher ratings were

- A question on ordering decimals
- A number operations question involving the deduction of related number facts
- A think of a number problem (elementary algebraic thinking)
- Reasoning in a money context

Self assessment comments

Note: In this section quotes are given a code with the initial of the student's institution and a number e.g. L23 is student 23 from the London Institute. The percentages given in brackets are percentages of the sample of 274 who chose to write comments.

The categorisation of the student comments (n=274) arose from the data itself, and the coding relied on interpretations which were checked during the process. This resulted in 9 categories, each with subcategories:

Category	Description	Number of students commenting
A	Level of confidence	103 (38%)
B	Assessment of knowledge	223 (81%)
C	Self-audit process	43 (16%)
D	Help used	20 (7%)
E	Remediation process	111 (41%)
F	Approaches to learning	16 (6%)
G	Miscellaneous	19 (7%)
H	Specific areas of difficulty	118 (43%)
J	Generic difficulties	100 (36%)

Given that the comments were spontaneous, revealing what the students felt was appropriate to communicate to their tutors, some of the numbers in the subcategories are small. This is not to say that some students did not have a view on these issues, rather that they did not choose to comment.

i) Confidence and Assessment of Knowledge

Although the numerical coding had asked students to rate each item in terms of how secure their response had been, the written comments revealed a distinction between their feelings of confidence and their assessment of mathematical knowledge. Of the students commenting on confidence, 27 (10% of whole sample) felt confident in all or most areas, 29 (11%) were not confident and 37 (14%) were confident that they could update ‘rusty areas’. Of those commenting on their knowledge, 110 (40%) felt they were rusty or out of date, and 98 (36%) felt their knowledge was patchy with 90 giving specific details. The 12 (5%) who felt they were weak or struggling included 2 who admitted to panicking. Typical responses from those students with gaps in their knowledge who were relatively confident of being able to remedy this were:

> I have not gone any further than GCSE level but found that all questions I attempted I remembered doing at some point in the past…I feel confident that I could pick up in all areas (D23)

> Had forgotten some formulae but understood what to do when I had checked them. Was basically OK but do feel like I need to do more revision and get more confidence. Do not feel this represents my general confidence level – been a long time since I did school maths so maybe it will all come back!! (L117)

One member of the smaller group who recognised their weaknesses and lack of confidence wrote:

> I really struggled completing this audit and feel very insecure about my subject knowledge (C30)

and another simply commented:

> Help!!! (D17)

ii) The self-audit process, help used and remediation in the future

The 43 students commenting about the process gave a variety of views. 9 (4%) said they had given up at some point or were unable to complete the self-audit, 11 (5%) expressed a negative emotional response and 8 (3%) a positive one:

> I attempted the first two sections of the audit and got to the point where I felt I wasn't getting anywhere. I do think I can do most of the last 4 sections I just haven't attempted them yet. (D10)

> Shape, space and measures frighten me ever since I was told I have a below-average spatial ability. (C32)

> when I first looked at the audit I thought I couldn't do it at all ~ but when I actually tried I could do it ~ and enjoy it (L44)

After doing the self-audit and consulting the specimen answers and commentary 23 (8%) felt that this was helpful, while others had consulted books, dictionaries and other resources. Only 4 (1%) appealed directly for help, without identifying what sort of help was necessary or how they might go about remediation themselves. Given the large numbers of students identifying their knowledge as rusty it is not surprising that a large number (80, 29%) said they needed to refresh, revise or practice and another 20 (7%) said they were already doing this.

iii) Approaches to learning

Some students were stimulated to comment about how they approached specific items and how they had learned in the past. Although only 16 (6%) made such comments, these revealed self-knowledge and reflection which could be helpful in their preparation for teaching:

> I tend to approach things slowly, building up from the bases I already know, which is probably good in the context of teaching at Primary level. (L38)

> I'm struggling sometimes to explain the theory behind my reasoning – I think this stems from being taught by rote for a lot of the basic mathematical functions (D31)
>
> I feel that my problem lies in my inability to see what type of Maths I should perform in order to answer the question rather than a failure to understand the methods. (C40)

iv) Areas of difficulty

The most common specific areas of difficulty identified were:

Area of mathematics	Number of comments	Percentage of sample
Graphs	38	14%
Shape and/or space	49	18%
Transformations	38	14%

The most common general areas of difficulty were:

Difficulty	Number of comments	Percentage of sample
Terminology	62	23%
Explaining strategy	30	11%

These comments accord with the items which received the lowest self ratings, and also support the interpretations offered earlier about the difficulties with terminology. Many of the questions asked the students to explain their methods or their reasoning, and this was clearly identified as a common difficulty.

v) Students with concerns

Weak students or students with negative feelings were a particular concern for the researchers. Looking at the rating scores for the whole sample of 432 and looking across categories at the comments made by 274 students it was possible to identify students who had low self ratings and/or were identifying themselves as concerned through their comments. In each cohort there were a group who expressed concern, but these did not necessarily have a low self rating. Similarly some students with low self ratings did not write any comment or chose not to express any particular concern. This is shown in the table below where a self rating of 50 would correspond to a mean rating of 2.4 per item.

Institution	Number expressing concern	Number expressing concern with self rating <50	Self rating <50 but not expressing concern
London	19	8	20
Cambridge	19	9	38
Durham	7	4	10
All	45 (10%)	21 (5%)	68 (16%)

Almost all the students who expressed concern had something positive to say. For instance one comment from a student with a low self rating expressed concern but was already thinking of doing something about it herself:

> I learned maths at school by rote. I have little or no understanding of <u>why</u> things are worked out in the way they are….It's also been 20 years since I attempted fractions/equations/long division etc. In short I'm having to start again in all areas. Three cheers for Derek Haylock! (C74, self rating of 34)

It was rare to have an entirely negative response. Many of this group expressed concern but remarked how helpful they found the commentary, or that they were only concerned about certain areas, or that they were prepared to put in extra effort to overcome their weaknesses. Coming back to mathematics after a gap was a common concern for this group of students. Many of them had either forgotten the mathematics or had never encountered some of the topics:

> The 'reasoning and proof' and 'probability' sections fill me with dread. I don't ever remember ever doing these type of 'maths' before and would dread having to teach this sort of thing to children. (D38, self rating 46)

However, one student with a very high self rating also expressed extreme concern about her confidence rather than her level of understanding:

> The fact that when I completed the questions I was shaking and had a headache suggests that I am less confident now than when I was doing my maths G.C.S.E.! I think that my understanding is fairly sound, but I need practice to improve my confidence. (H34, self rating 77).

Performance in the audit

The audit consisted of 16 items, each marked on a 0-4 scale as follows:

0-not attempted, no progress towards a final solution
1-insecure, partial solution, incorrect
2-secure in parts, insecure in parts
3-secure, small errors, explanations acceptable but not completely convincing
4-completely secure with convincing and rigorous explanations (not necessarily using algebra)

In general, those with low self ratings had relatively low scores on the final audit and those with high self ratings had relatively high scores on the audit. Further discussion of the problems which persisted and the new ones which presented themselves will be found in the conclusion.

(a) Questions with low mean scores

The questions with the lowest mean scores involved

- A shape and space question involving transformations
- Two questions on reasoning and proof
- A question involving perimeter, area and Pythagoras

(b) Questions with high mean scores

The questions with the highest mean scores involved

- Ordering decimals
- Equivalences between fractions, decimals and percentages
- Ordering fractions
- Providing a strategy for finding the general term of a sequence

CONCLUSION

The items which students identified as particularly difficult on the self-audit did not correspond to those areas of weakness we had identified in the earlier research. In particular, the items on reasoning and proof did not have particularly low mean ratings and were not raised very much in the students' comments. In the self-audit they were more preoccupied with shape and space, and with the equations and graphs of straight lines. Are these to be taken seriously or were difficulties with terminology getting in the way? Many students did not know the terminology of associativity and commutativity and this almost certainly accounted for the difficulty with the number operations question. Similarly the terminology of transformations may have accounted for difficulties with one of the Shape and Space questions. In the graph question the word gradient may have been the problem but it also seems likely that seeing the connection between the graph and the equation was a source of difficulty. The fourth item with a low mean rating involved constructing a triangle, using the converse of Pythagoras, and the ratios of lengths and areas in similar shapes. Students did not specify what the difficulty was here, but the complexity of this item may well have been the problem.

In the formal audit, the two low scoring items on reasoning and proof accorded with the previous research. Perhaps students had not addressed reasoning and proof adequately because they had not identified this area as problematic in the self-audit. Perhaps they were more probing questions. The item on transformations similar to that on the self-audit also had a low score, even though students had identified this and the associated terminology as difficult. The fourth low scoring item involved Pythagoras, which was also involved in the fourth low rated item on the self-audit, although this was not mentioned explicitly in self assessments. The difficulties with terminology in the number operations, identified by students on the self-audit, did not seem to cause so many problems on the formal audit, and the graph problem was tackled more successfully when set in a ‘real life’ context. In both cases this later success may have been a feature of students’ improved understanding or a feature of the item itself. This is a very mixed picture. In some cases, self assessed difficulties seem to have been resolved and in others they persisted. It is worth noting that in both the self-audit and the audit, items involving number concepts and operations were generally very secure. This may reflect strengths in number topics, the attention given to number in course teaching, or the degree of difficulty in the items themselves.

This comparison between the self-audit and the audit raises many questions. Previous weaknesses in reasoning and proof have been confirmed by the audit, and both the self-audit and the audit highlight new difficulties in shape and space. Just how seriously should these be taken? Are these weaknesses features of the assessment items and the terminology used in them or indications of shaky understanding which does not stand too much probing? Are some of these weaknesses irrelevant for prospective primary teachers? We would argue that the items on reasoning and proof in the audit demanded very little technical expertise but they did require the ability and willingness to investigate a situation, look for general patterns, make conjectures and try to justify them i.e. expertise in syntactic knowledge. This change in orientation may be too much to achieve in the one year PGCE course. The weaknesses in the shape and space items running through both the self-audit and the audit involved Pythagoras and transformations, which could both be regarded as elements of substantive knowledge. Of the two, the difficulties with transformations (translation, reflection, rotation, enlargement) would be of more concern for primary teachers.

Most of the students who commented about their confidence were either confident in all or most areas or were confident that they could update their knowledge. In terms of knowledge, most felt rusty or out of date, or felt that their knowledge was patchy with some strong and some weak areas. Of those who commented on what they needed, the majority acknowledged the need for revision but fewer said exactly how they intended to go about it. It seems likely that many were relying on course provision between the self-audit and the audit. In specifying difficulties, there were common patterns in the specific items identified and in the generic difficulties across items. This was helpful to tutors in course provision between the self-audit and the audit, but it is not clear if the students acted upon their own self assessments. A small group reflected more generally on how they had learned mathematics and how they approached it now; these insights may have been helpful to them in putting themselves back in the shoes of the learner.

There was an identifiable but small group (10%) of students for whom the self-audit raised particular concerns. Their responses were characterised by emotional language and sometimes reflected negative learning experiences in the past. Half of these also had low self ratings, but there were also students with high self ratings who expressed concern. Similarly there were students with low ratings who expressed no concern. Being able to express their concern and articulate their fears at this stage in the process may have been helpful to these students, since tutors were then sensitised to them and could respond accordingly. The choice of peer support groups and peer tutors was made with these considerations in mind.

The use of specific items to focus students' self assessment of their subject knowledge is useful in that it encourages them to see areas of strength as well as weakness. However, the degree to which understanding is probed depends on the items themselves and students may be lulled into a false sense of security by success on relatively straightforward items. Moreover, identifying weaknesses does not necessarily mean that students will then start to address them independently. They have a lot to do in a busy PGCE year and will almost certainly need to rely on focused tutor input and/or structured peer support. The fact that so many students seem to take a fairly sanguine view of the whole process is encouraging although some of them may be too complacent. The identifiable group with particular worries is still of concern. Voicing their concern may have been helpful to these students and it continues to remind tutors of the need for sensitivity when handling mathematical subject knowledge. In the same way that aspects of substantive and syntactic knowledge may take longer than one year to develop, developing confidence and positive attitudes will almost certainly be a long term project for these students.

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