PROSPECTIVE PRIMARY TEACHERS' MATHEMATICS SUBJECT KNOWLEDGE: SUBSTANCE AND CONSEQUENCE

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In the context of the day symposium on prospective primary teachers' mathematics subject knowledge held at the Birmingham day conference, this paper summarises some findings from research at the Institute of Education. These findings have provided part of the backdrop to subsequent collaborative work with researchers in Cambridge and York.

INTRODUCTION

The purpose of this paper is to present a succinct update on aspects of research reported at previous meetings of BSRLM (Rowland *et al*., 1998, 1999) arising from research on primary PGCE trainees' mathematics subject knowledge at the Institute of Education, University of London. Government Circular 4/98 charged Initial Teacher Training 'providers' with the audit and remediation of students' subject matter knowledge (SMK). The Circular required providers to "audit trainees' knowledge and understanding of […] mathematics …Where gaps in trainees' subject knowledge are identified, providers of ITT must make arrangements to ensure that trainees gain that knowledge during the course" (DfEE, 1998, p. 48). Whilst we shared the widespread resentment at this interference in our management of subject matter issues, our research on prospective primary teachers' mathematics subject knowledge has undeniably been both motivated and facilitated by it. The conceptualisation of subject knowledge which informed the project and its relation to teaching has been detailed extensively elsewhere (Goulding, Rowland and Barber, 2002), and is summarised in Maria Goulding's paper in these proceedings. It includes the key concepts of substantive and syntactic knowledge (Schwab, 1978) as components of subject matter knowledge. Substantive knowledge concerns the key facts, concepts, principles and explanatory frameworks in a discipline, whereas syntactic knowledge is more process oriented, concerning the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community.

In this paper, we describe our approach to the audit of the mathematics SMK of 173 primary trainees in 1998-99. This was the first cohort of students following the one-year PGCE course to whom the requirements of Circular 4/98 applied by statute. We had, however, piloted the audit and a draft version of the 'standards' on a voluntary basis the previous year, and draw on an analysis of one audit item in this paper.

CONTEXT

The structure of the primary PGCE under consideration is such that by the middle of January, with fully six months of the course remaining, the main content areas – number concepts and operations, data handling, mathematical processes, shape and space, measures, algebra, probability – have been ‘covered’ in lectures and workshops. A 90-minute written assessment consisting of 16 test items in mathematics is administered at this point of the course. Each trainee’s response to each question includes a self-assessment of their ability to tackle it.

The course includes two extended placements in schools in the latter parts of the second and third terms. Given these and other demands of the course, the major SMK remediation opportunity comes between the first and second placements.

Other aspects of the taught course based in the University	Maths SMK audit	Other aspects of the taught course based in the University	School placement 1	SMK peer-tutoring and remediation (with other aspects of the course)	School placement 2
Term 1 (autumn)	Term 2 (spring)			Term 3 (summer)	

Table 1: The chronology of the PGCE course

During school placements, each student works under the joint supervision of a school-based mentor and a university tutor. The two supervisors agreed on assessments of the student’s performance in teaching mathematics towards the end of (and in the context of) each placement, against the standards of Circular 4/98.

TRAINEES’ MATHEMATICAL THINKING: GENERALISATION AND PROOF

One dimension of our research was to identify what mathematics (within the remit of Circular 4/98) primary trainees find difficult, and the nature of their errors and misconceptions in these areas. In this brief account, we focus on just two aspects of the trainees mathematical thinking.

Generalisation

Just over half the trainees were insecure in an item designed to address the ability to observe and express a generalisation (see Rowland *et al* 2000 for details).

> Check that 3+4+5=3x4 8+9+10=3x9 29+30+31=3x30
>
> Write down a statement (in prose English) which generalises from these three examples. Express your generalisation using symbolic (algebraic) notation.

Some students did not recognise the features common to all three examples and tried to derive a generality from the first case only. For example, one wrote:

> Three consecutive numbers added together equals the product of the first two numbers.
> $n + (n+1) + (n+2) = n \times (n+1)$.

Others were able to express the generality in their own words but not symbolically. Responses such as $a + b + c = 3b$ captured part of the picture but omitted the essential condition that the three numbers being summed are consecutive (or in arithmetic sequence).

A few students struggled to find the words to communicate what they could ‘see’ in the examples. One wrote:

> Three ascending numbers may be equal, in sum, to 2 numbers that are multiplied together. The middle number of the sequence and the over all numbers are multiplied to give the same answer as those added together.

This item, then, exposed weaknesses in recognising and articulating pattern and relationships, identifying significant elements, and in formulating expressions to represent these relationships. This relates to syntactical subject knowledge, since inductive reasoning is central to the philogeny and the otogeny of mathematical knowledge. It is perhaps not difficult to share the concern of the UK government about prospective primary school teachers who, for example, find it so difficult to perceive and communicate unity of form (let alone of meaning) in the three equations. At the same time, we would question the adequacy of “guided self-study” (DfEE, 1998, p. 48) in the face of such cognitive obstacles.

Proof

Currently, there is evidence for concern in the UK about students’ facility with mathematical proof, both at school and at university level (see e.g. London Mathematical Society, 1995). One argument suggests that logical reasoning was a casualty of curriculum and assessment reforms in the 1970s and 1980s. Circular 4/98 requires that trainees demonstrate “that they know and understand […] methods of proof, including simple deductive proof, proof by exhaustion and disproof by counter-example (DfEE, 1998, p. 62)”. The following item was designed to audit this ‘standard’.

> A rectangle is made by fitting together 120 square tiles, each 1 cm^2. For example, it could be 10cm by 12 cm. State whether each of the following three statements is true or false for every such rectangle. Justify each of your claims in an appropriate way:
>
> (a) The perimeter (in cm) of the rectangle is an even number.
>
> (b) The perimeter (in cm) of the rectangle is a multiple of 4.
>
> (c) The rectangle is not a square.

More than one mode of justification is possible for each part, but we anticipated some deductive arguments for (a), counterexamples for (b), and perhaps contradiction ($\sqrt{120}$ is not an integer) for (c). In the event, only one third of students made a secure response to the whole question and 30% either gave insecure answers to all three parts or did not attempt the question (see Rowland *et al.*, 2001 for a detailed analysis). Most interesting, perhaps, is the fact that a significant number of students did not seem to perceive the second statement as amenable to personal investigation on their part, which (for those who did so) uncovers counterexamples to refute the statement. Some claimed, for example, that the statement must be true because 120 is a multiple of 4, or even because the rectangles have 4 sides. These prospective teachers evidence little or no sense of mathematics as an experimental test-bed, in which they might confidently respond to an unexpected student question “I don’t know, let’s find out”.

SUBJECT KNOWLEDGE AND CLASSROOM PERFORMANCE

We move on now to data that have enabled us to build on and update our earlier findings (Rowland *et al*., 2000) associated with another of our project goals – investigating the relation between trainees’ SMK and their teaching competence. The level of each student’s subject knowledge (based on the audit) was categorised as low, medium or high, corresponding to the need for significant remedial support, modest support (or self-remediation), or none. In addition, assessments of the students’ teaching of mathematics were made (against the standards set out in Circular 4/98) on a three-point scale weak/capable/strong. For the 1998-99 cohort, these assessments were made (a) on both first and second placements, and (b) with respect to both ‘preactive’ (related to planning and self-evaluation) and ‘interactive’ (related to the management of the lesson in progress) aspects of mathematics teaching (following Jackson, 1966 and Bennett and Turner-Bisset, 1993). Tables 2 to 5 below show the four 3 by 3 contingency tables, for Placement 1 (N=167: six students had withdrawn from the course) and Placement 2 (N=164: three more students had withdrawn), together with expected frequencies (in parentheses) based on the null hypothesis that audit performance and teaching performance are independent.

SUBJECT KNOWLEDGE AUDIT	TEACHING PRACTICE PERFORMANCE		
	Strong	Capable	Weak
High	**17 (12.4)**	**16 (17.3)**	**1 (4.3)**
Middle	**31 (29.2)**	**38 (40.7)**	**11 (10.1)**
Low	**13 (19.4)**	**31 (27.0)**	**9 (6.7)**

Table 2: Placement 1, preactive

SUBJECT KNOWLEDGE AUDIT	TEACHING PRACTICE PERFORMANCE		
	1 (strong)	2 (capable)	3 (weak)
A (high)	**18 (12.4)**	**12 (16.1)**	**4 (5.5)**
B (middle)	**32 (29.2)**	**37 (37.8)**	**11 (12.9)**
C (low)	**11 (19.4)**	**30 (25.1)**	**12 (8.6)**

Table 3: Placement 1, interactive

SUBJECT KNOWLEDGE AUDIT	TEACHING PRACTICE PERFORMANCE		
	Strong	Capable	Weak
High	**12 (8.1)**	**18 (14.1)**	**4 (11.8)**
Middle	**20 (18.5)**	**33 (32.3)**	**25 (27.1)**
Low	**7 (12.4)**	**17 (21.6)**	**28 (18.1)**

Table 4: Placement 2, preactive

SUBJECT KNOWLEDGE AUDIT	TEACHING PRACTICE PERFORMANCE		
	1 (strong)	2 (capable)	3 (weak)
A (high)	**13 (8.5)**	**19 (18.2)**	**2 (7.3)**
B (middle)	**21 (19.5)**	**42 (41.9)**	**15 (16.6)**
C (low)	**7 (13.0)**	**27 (27.9)**	**18 (11.1)**

Table 5: Placement 2, interactive

Each table has df=4, and values of χ^2 less than 9.5 support the null hypothesis against the alternative that audit performance and teaching performance are in some way linked ($p<0.05$). The χ^2 values for the preactive and interactive data are 8.2 ($p=0.085$) and 10.5 ($p=0.03$) respectively for Placement 1 and 17.8 ($p=0.002$) and 13.6 ($p=0.009$) for Placement 2. Thus, the association between audit score and teaching performance is significant for three of the four analyses, the exception being the preactive dimension of the first placement. For the moment, we conjecture that the assessment of preactive aspects favours not only clarity about mathematics teaching and learning, but also a certain kind of bureaucratic competence that acts as some kind of 'leveller' in the very first exposure to work in schools. Taken together, however, these results support our earlier findings with the 1997-98 cohort (Rowland *et al.*, 2000) and point to the positive effect of strong SMK in both the planning and the 'delivery' of elementary mathematics teaching.

CONCLUSION

We have drawn attention to the problematic nature of generalisation and proof as a component of the mathematics SMK of pre-service elementary teachers, adding further weight to the doubts of Goulding and Suggate (2001) that much can be done to remedy trainees' difficulties with proof within initial training, especially given the multiple demands on them in all areas of the curriculum in an intensely pressured course. We would expect that clarity of understanding of the nature of proof and refutation in mathematics would inform the trainees' approach to questioning and enquiry with their students, and we are struck by the robustness under replication of our earlier finding (Rowland *et al.*, 2000) that effective classroom teaching of elementary mathematics is associated with secure SMK at a level beyond the elementary curriculum. It may be that, even within the constraints of PGCE courses, greater priority could be given to syntactic dimensions of SMK, although inevitably this would be at the expense of substantive elements.

The link between secure subject knowledge, as measured by the audit, and competence in the classroom might be thought to be no great surprise. There were, of course, a few salutary exceptions to the rule (the north-east and south-west cells of tables 2 to 5). The paper by Rowland, Huckstep and Thwaites in this volume is a contribution to understanding ways in which subject knowledge comes into play in the classroom in the teaching of elementary mathematics.

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