

# THE CHOICE OF EXAMPLES IN THE TEACHING OF MATHEMATICS: WHAT DO WE TELL THE TRAINEES?

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*The mathematics subject matter knowledge of primary school teachers has in recent years become a high profile issue in the UK and beyond. This paper reports on one dimension of a videotape study of mathematics lessons prepared and conducted by primary PGCE trainees. The aim of the study was to identify ways in which their subject knowledge, or the lack of it, was evident in their teaching. One significant issue that emerged was the particular examples chosen and used within the lessons. This raises questions about what advice we should be giving such novice teachers in their training.*

## INTRODUCTION

The seminal work of Lee Shulman conceptualises the diversity of the knowledge required for teaching. His seven categories of teacher knowledge include three with an explicit focus on ‘content’ knowledge: subject matter knowledge, pedagogical content knowledge and curricular knowledge. Shulman (1986) notes that the ways of discussing subject matter knowledge (SMK) will be different for different subject matter areas, but adds to his generic account Schwab’s (1978) notions of substantive knowledge (the key facts, concepts, principles and explanatory frameworks in a discipline) and syntactic knowledge (the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community). For Shulman, pedagogical content knowledge (PCK) consists of “the ways of representing the subject which makes it comprehensible to others...[it] also includes an understanding of what makes the learning of specific topics easy or difficult ... (Shulman, 1986, p. 9). PCK is particularly difficult to define and characterise, but seems essentially to conceptualise the hitherto missing link between knowing something for oneself and being able to enable others to know it.

In its Circular 4/98, the UK government specified for the first time a curriculum for Initial Teacher Training (ITT) in England (DfEE, 1998), setting out what was deemed to be the “knowledge and understanding of mathematics that trainees need in order to underpin effective teaching of mathematics at primary level”. This paper is one outcome of ongoing collaborative study, motivated at first by Circular 4/98, of prospective primary teachers’ mathematics subject knowledge between researchers at the universities of Cambridge, London, Durham and York under the acronym *SKIMA* (subject knowledge in mathematics). The conceptualisation of subject knowledge and its relation to teaching which informed the project has been detailed extensively elsewhere (Goulding, Rowland and Barber, 2002). The focus of the research reported in this paper is on ways that trainees’ mathematics content knowledge can be observed to ‘play out’ in practical teaching during school-based placements.

## METHOD

This study took place in the context of a one-year (three term), full-time Post-Graduate Certificate in Education course for prospective primary school teachers in a university faculty of education. Each of the 149 trainees followed a route focusing either on the 'lower primary (LP)' years (ages 3-8) or the 'upper primary (UP)' (ages 7-11). About one month into the second term of the course, a 16-item audit instrument was administered to all the trainees, under semi-formal conditions (more details are given in Goulding *et al.*, 2002). Their responses were used to identify groups of trainees with 'high', 'medium' and 'low' scores. Two trainees from each subject knowledge category and within the two LP/UP groups were chosen for observation. Two mathematics lessons taught by each of the trainees were observed and videotaped i.e. 24 lessons in total. These took place approximately in the 5th and 7th weeks of the 8-week placement; school half term occurred between the two observed lessons. Trainees were asked to provide a copy of their planning for the observed lesson. As soon as possible after the lesson (usually the same day) the observer/researcher wrote a *Descriptive Synopsis* of the lesson. This was a brief (4-500 words) account of what happened in the lesson, so that a reader might immediately be able to contextualise subsequent discussion of any events within it. From that point, we took a grounded approach to the data for the purpose of generating theory (Glaser and Strauss, 1987). In this way, we compared our interpretations of episodes from the 24 videotaped lessons. In particular, we identified aspects of trainees' actions in the classroom that seemed to be significant in the limited sense that it could be construed to be informed by the trainee's knowledge of *mathematics* subject matter knowledge or *mathematics* pedagogy as opposed to other more general kinds of pedagogical awareness or expertise. Next, we elaborated the *Descriptive Synopsis* into an *Analytical Account* of each lesson. In these accounts, significant moments and episodes were identified and coded, with appropriate justification and analysis. We focus now on just one of the 18 codes identified - the trainees' *choice of examples* in their teaching.

## CHOICE OF EXAMPLES

We shall begin with some brief consideration of the ways that examples are used in teaching, before giving illustrations from the lessons observed. From these, we isolate particular pitfalls that indicate the kind of guidance we might usefully give these trainees before and during their placements.

### **The place of examples in mathematical teaching**

Reflecting on what it is that learners gain from examples, it is helpful to distinguish two rather different uses of examples in teaching. The first is essentially inductive - providing (or motivating students to provide) examples *of* something. The 'something' is *general* in character (e.g. the notion of line symmetry, or the fact that the sum of two odd integers is even); the examples are *particular* instances of the generality. The use of examples to embody abstract concepts and to general procedures is commonplace pedagogical practice. Thus, we teach a (general) procedure by a (particular) performance of that procedure. For example, if we set out to teach subtraction by decomposition, we

might perform, say, 62-38 in column format. It is important to note that the 6, the 2, the 3 and the 8 in the previous sentence were all *chosen*, and with some care. The range of such choices available to the teacher is usefully articulated in terms of what Marton and Booth (1997) call ‘dimensions of variation’<sup>1</sup>. This notion extends to many, if not all, kinds of learning. In essence, we learn from discerning variation, and what varies in our experience determines what we learn. The provision of examples must therefore take into account the dimensions of variation inherent in the objects of attention. What students learn about two-digit column subtraction will depend, significantly if not entirely, upon the teacher’s awareness of the possibilities for variation within that context, and the choices s/he makes in consequence.

In the case of concepts, the role of examples is to provoke or facilitate abstraction: once a set of examples has been unified by the formation *of* a concept, subsequent examples can be assimilated *by* the concept (Skemp, 1979). When a concept has been formed and named by an individual, s/he is able to entertain examples of it outside the realm of personal experience. A teacher’s choice of examples for the purpose of abstraction will reflect his/her awareness of the nature of the concept and the category of things that it comprehends, and the dimensions of variation within that category.

The second use of examples in teaching, more often called ‘exercises’, is not inductive, but illustrative and *practice*-oriented. We note here that exercises are examples, selected from a class of possible such examples. In the case of two-digit subtraction, 20 exercises might be chosen from the class of some 4000 possible examples. Characteristically, having learned a procedure, the student rehearses it on several such ‘exercise’ examples. This is initially to assist retention of the procedure by repetition, then to develop fluency with it. Such exercises are also, invariably, an instrument for assessment, from the teacher’s perspective. Again, the selection of such examples by teachers is neither trivial nor arbitrary. The argument for examples to be ‘graded’ is generally well understood, so that students experience success with routine examples before trying more challenging ones. Bierhoff (1996) has commented that English primary textbooks are poor examples of pedagogy in their provision of examples, compared with those in Germany and Switzerland, whose authors demonstrate far greater didactic awareness.

In both senses of the word, we suggest that the examples provided by a teacher ought, ideally, to be the outcome of a reflective process of *choice*, a *deliberate* and informed selection from the available options, some ‘better’ than others in the sense illustrated above. While we do not pretend to be able to infer such a process of choice, or the lack of it, from the evidence of the videotapes, we can comment on the examples actually chosen by trainees, and how they compare with available alternatives.

### **The Trainees’ Choice of Examples**

Whilst we looked for instances of both good and poor choices of examples, the latter seemed to be more prevalent. To redress the balance of this section, therefore, we begin

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<sup>1</sup> I am grateful to Anne Watson and John Mason for bringing the work of Ferenc Marton to my attention.

by citing a somewhat isolated (though not unique) instance that seems to us to draw on some key aspects of mathematics content knowledge.

Naomi's lesson was with a Year 1 class. The main activity<sup>2</sup> was about the meaning of the 'difference' of two numbers within 20. Naomi had achieved the maximum score on the audit, yet there is little evidence of overt SMK in the lesson. The lesson would be rich material for a case study, but this is not our purpose here. Suffice it to note the following episode from the mental and oral starter, where the children practised bonds to 10. They sat in a circle, and Naomi chose particular individuals to answer questions such as "If we have nine, how many more to make 10?" The sequence of starting numbers was 8, 5, 7, 4, 10, 8, 2, 1, 7, 3. This seems to us to be a good sequence, for the following reason. The first and third numbers are themselves close to 10, and require little or no counting. 5 evokes a well-known double - doubling being an explicit NNS strategy. The choice of 4 seemed (from the videotape) to be tailored to one of the more fluent children. The degenerate case  $10+0$  merits the children's attention. One wonders, at first, why Naomi then returned to 8. The child (Bill) rapidly answers '2'. The answer to our question becomes apparent when Naomi asks the next child, Owen, what he must add to 2 to make 10. Owen counts from 2 on his fingers, and declares '8'. Naomi then asks Owen to recall and reflect on Bill's question, and his answer. There seems to be some *conscious design* at play in Naomi's sequence. Her choice of examples (a) was at first 'graded' (b) included later an unusual/degenerate case, and (c) finally highlighted a key structural property of addition i.e. commutativity. She draws attention to this relationship yet again in her final choice of 7, then 3, and in her comments on this pair of examples.

Certain key categories of the trainees' less felicitous choices of examples is beginning to emerge. One is the choice of examples that *obscure the role of the variables* within it. One such case concerns Michael, in a lesson with a Year 4 class. The main activity was about telling the time with analogue and digital clocks. One group was having difficulty with analogue quarter past, half past and quarter to. Michael intervened with this group, exemplifying half past with half past six. When asked to show half past seven on their clocks, one child put both hands on the 7.

Another instance took place in a Year 6 lesson which began with work on co-ordinates. Kirsty asked the children to identify the co-ordinates of points as she marked them on a grid. She reminds them that "the  $x$ -axis goes first". Her first example is the point (1, 1), which is clearly ineffective in assessing the children's grasp of the significance of the order.

Several other similar examples were readily identified in the videotaped lessons, where the role of a particular variable in a calculation is obscured by the presence of another

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<sup>2</sup> The National Numeracy Strategy *Framework* (DfEE, 1999) guidance effectively segments each mathematics lesson into three distinctive and readily-identifiable phases: the *mental and oral starter*, the *main activity* (an *introduction* followed by *groupwork*) and the concluding *plenary*.

variable with the *same* value. Chloe is reviewing with a Year 1/2 class a strategy for adding and subtracting 9 and 11. She asks one child to demonstrate on a 1-100 number square by adding 9 to ... 9. To criticise her choice of starting number (9) may seem somewhat churlish. But it was the *first* example offered in the lesson, and she had some 90 starting numbers to choose from - some of which would be unsuitable for other reasons. For example, Chloe later demonstrates a particular strategy for subtracting 19 (subtracting 20 and adding 1) on the number square. The visual representation would be 'up two, right one', like a knight's move. Chloe chooses 70 for the starting number in her first example, on the extreme right boundary of the 1 to 100 square. After moving up two squares to 50, there is no 'right one' square: it is then necessary to move down and to the extreme left of the next row.

A second category of poor choices of examples arises from the selection of calculations to illustrate a particular procedure, when *another procedure would be more sensible* to perform those particular calculations. A minor instance occurred in Naomi's lesson on 'difference', where she asked (on a worksheet) for the difference between 11 and 10, expecting them to 'count on' from the lesser of the two numbers. This is akin to giving e.g. 302-299 in a set of exercises on subtraction by decomposition. A more worrisome case concerned Laura's choice of demonstration examples in her first videotaped lesson, on column multiplication (the standard 2-digit by 1-digit algorithm) with a Year 5 class. Her first example ( $37 \times 9$ ) is not a bad one (though not the best either), but she then goes on to work through  $49 \times 4$ ,  $49 \times 8$  and  $19 \times 4$ . Now, the NNS emphasises the importance of *mental* methods, where possible, and also the importance of choosing the most suitable strategy for any particular calculation.  $49 \times 4$ ,  $49 \times 8$  and  $19 \times 4$  can all be more efficiently performed by rounding up, multiplication and compensation e.g.  $49 \times 4 = (50 \times 4) - 4$ . For that matter,  $49 \times 8$  is readily found by doubling the answer to  $49 \times 4$ . As we mentioned earlier, the NNS makes much of doubling strategies, and  $19 \times 4$  could be a double-double. In any case, to carry out these calculations by column multiplication flies in the face of any messages about selecting 'sensible' strategies.<sup>3</sup>

Finally, we note that the videotapes offer copious instances of *examples being randomly generated*, typically by dice. This may have a limited but useful place in the generation of practice exercises, but it is pedagogically perilous in the teaching of procedures or concepts, when, as we have argued, it is simply not the case that any example is as good as any other. Working with a Reception class on subtracting numbers from 10, Colin generated his expository examples - 5, 3, 8, in that order - with a specially modified die. This contrasts with Naomi's skilful control of the examples in the episode described above, with a closely-related learning objective (bonds to 10). There seems to be some confusion in the minds of many trainees between the legitimate random choice of examples to enhance conviction about the truth of some principle or the efficacy of some established procedure on the one hand, and the choice of examples to inculcate

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<sup>3</sup> At the Oxford session, Alison Price pointed out, in the episode with Chloe, that children would previously have been encouraged to memorise  $9+9$  as a 'double', and that, for that reason, this was not a good example from which to teach a general strategy for adding 9.

awareness of a procedure or concept in the first place on the other. The latter is often better controlled and determined by the teacher, and random selection of examples in this case is effectively an abdication of responsibility.

## CONCLUSIONS

Our grounded approach to the analysis of the lessons has highlighted several normative notions of teaching practice, one of which we have elaborated in this paper. This aspect - the teacher's *choice of examples* - was noticeably prevalent in the *Analytic Accounts* of the 24 lessons. The evidence from our research has greatly enhanced our own awareness that novice teachers need guidance and help in appreciating the different roles of examples in mathematics teaching, and the existence of some common pitfalls in the selection of examples. In particular we have isolated issues concerning:

examples that *obscure the role of the variables* within it;

examples intended to illustrate a particular procedure, for which *another procedure would be more sensible*; and

examples for instruction (as opposed to exercise examples) being *randomly generated*, typically by dice, at a point when it would be preferable for the teacher to be making careful choices.

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